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THERE IS A CONTINUUM AMBIGUITY FOR ELASTIC πN AMPLITUDES

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The implicit-function method of constructing phase-factor continuum ambiguities in phase-shift analysis is briefly reviewed, and new numerical examples are given of ambiguities in πN phase shifts at 1997 MeV. Since the ambiguous amplitudes differ by more than 5%, while the corresponding cross sections and polarizations are equal, to better than a computational accuracy of 0.007%, numerical credence is given to the theoretical claim that the continuum ambiguity exists.

The continuum ambiguity, which generally exists in the determination of the scattering amplitudes of an elastic scattering process above the inelastic threshold, was the subject of theoretical [1] and numerical [2] investigations during the 1970's. There was general consensus [3] that an inherent ambiguity *does* exist in energy-independent phase-shift analysis, so that energy-dependent analyses, for example those of the Höhler group [4], in which analytic properties of the amplitudes were used, are in most cases necessary. Nevertheless there has been a claim [5] that there is no effective ambiguity in energy-independent isospin $\frac{3}{2}$ pion–nucleon phase-shift analysis below 2 GeV (CMS), and in particular that the implicit-function method [1] does not yield a true continuum ambiguity.

In earlier work, small changes in the measurables (cross sections and polarizations) were tolerated in the interests of computer time, on the grounds that they are in any case subject to experimental errors. Evidently the authors of ref. [5] have been misled by this fact into believing that the origin of the ambiguities in our amplitudes was the lack of precision in holding observables fixed.

It is the purpose of this letter, after directing the reader to the mathematical literature, so that she or he can judge it on its merits, to present new ambiguities, based on the 1997 MeV SACLAY data [2], in which the π^+p data are kept to within 10^{-5} accuracy (ridiculously small for experimental purposes), and the variations of the phases of the amplitudes for the ambiguities are in sone cases as large as 10%–35%. For comparison a Wolfenstein parameter has been sketched for the various ambiguities (see below). Despite the size of the ambiguities, it should be remarked that the strength of inelastic unitarity in limiting the otherwise arbitrary angle-dependent phase of a physical quantity is quite impressive. A comparison of the predictions made in the 1976 continuum ambiguity analysis [2], and the status of the Δ resonances according to the 1982 Particle Data Group listings [6] adequately vindicates the relevance of the approach.

The basic idea of the implicit-function method is to write the partial-wave inelastic unitarity relation,

$$A_l = A_l^2 + D_l^2 + I_l, \tag{1}$$

where D_l and A_l are respectively the real and imaginary parts of the partial-wave amplitudes, and where $I_l = \frac{1}{4}(1-\eta_l^2)$ is the inelasticity (constrained to lie in the interval $[0, \frac{1}{4}]$), in such a way that D_l is an implicit function of A_l and the experimentally measured differential cross section $(\mathrm{d}\sigma/\mathrm{d}\Omega)$ and polarization (P). Thus (1) is construed as a functional equation,

$$A_l = \Phi_l(A, \mathrm{d}\sigma/\mathrm{d}\Omega, P, I). \tag{2}$$

The mathematical part of the work [7] consists in showing that the function Φ_l has suitable properties,

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such that, if A_l and I_l satisfy (2), then if we change I_l, the inelasticity, by a small increment ΔI_l, a corresponding increment ΔA_l exists, such that

$$A_l + \Delta A_l = \Phi_l(A + \Delta A, \mathrm{d}\sigma/\mathrm{d}\Omega, P, I + \Delta I). \tag{3}$$

Note that $\mathrm{d}\sigma/\mathrm{d}\Omega$ and P are unchanged, so we have generated an "ambiguity"; and since the ΔI_l may be chosen continuously, it is continuum ambiguity.

One difficulty is that, when one alters I_l, the A_l are changed according to (3), and one may generate a "physically" unacceptable tail of high partial waves, an accusation that has been levelled [5]. Indeed, in the initial work [8], in which no continuation into the complex $\cos\theta$-plane was made, the objecction can be sustained. However, in later work [1], analyticity in the $\cos\theta$-plane was guaranteed, in the spin 0–spin $\frac{1}{2}$ case most elegantly by using the Barrelet formalism [9], so that a tail of partial waves lying within the correct exponential bound is automatically generated. Any claim that the decrease in partial waves should be faster than this is unfounded, and should have no place in an unbiased analysis.

A complication is that zeros of a certain function related to the real part of the amplitude [1], if they occur within the small Martin ellipse in the $\cos\theta$-plane, or the corresponding annulus in the Barrelet variable, lead to constraints on the ways in which the inelasticities may be changed. For every simple zero of this function in the annulus, ΔI_l in (3) is constrained for one l-value; and one additional l-value is constrained in order to keep the total cross section σ_{tot}, which is experimentally measured, fixed. Hence we separate the I_l into a constrained set I^{c}, and a free set I^{f}, replacing (3) by

$$A_l + \Delta A_l = \Phi_l(A + \Delta A, \mathrm{d}\sigma/\mathrm{d}\Omega, P, \sigma_{\mathrm{tot}}, I^{\mathrm{c}} + \Delta I^{\mathrm{c}}, I^{\mathrm{f}} + \Delta I^{\mathrm{f}}). \tag{4}$$

Here ΔI_l^{f} are chosen freely, $\mathrm{d}\sigma/\mathrm{d}\Omega, P$ and σ_{tot} are held fixed, while ΔA_l and ΔI_l^{c} are determined by (4).

The limits of the continuum ambiguity are determined by the requirement that all the I_l lie in the interval $[0,\frac{1}{4}]$. We can choose the ΔI_l^{f} such that the I_l^{f} never violate this requirement; but the ΔI_l^{c} are not under our control. The moment that one I_l^{c} leaves the allowed interval, the ambiguity terminates in that direction. The crucial point is that the number of the I^{c} is finite, being equal to the number of zeros of the function related to the real part of the amplitude inside the small Martin ellipse, plus one for σ_{tot}, so one can be sure whether or not the unitarity requirement is satisfied. Usually one changes only one I_l^{f} at a time, and of course one ensures that $I_l^{\mathrm{f}} + \Delta I_l^{\mathrm{f}} \in [0,\frac{1}{4}]$.

In practice, the system (4) is converted into a Newton iteration for numerical work, and the unitarity condition on the finite set I_l^{c} can be easily checked. This is the main advantage of the implicit-function method over the procedure whereby one simply generates a new ambiguity by multiplying the amplitudes by a phase-factor [3]. This certainly leaves the measured quantities unchanged; but one then has in principle to check an infinite number of new inelasticities I_l, to see if the unitarity condition is still respected. One can never be sure.

In the Newton iteration, when convergence is achieved, and this involves a good choice of the partial waves in I^{c}, and a good choice of the free inelasticity to be changed, then the final values of $\mathrm{d}\sigma/\mathrm{d}\Omega$, P, and σ_{tot} are essentially the same as their initial values. Small changes are caused by limitations of machine accuracy, as well as the choice of the point at which the partial-wave series is truncated and the criterion that signals convergence of the iteration. In earlier work, relatively small changes of the measured quantities were tolerated, on the grounds that these are in any case subject to experimental error. However, in view of the doubts cast on the ability of the method to generate true "theoretical" continuum ambiguities, we undertake below to generate substantial changes in the amplitudes, with completely insubstantial changes in the observables.

Because of the large number of degrees of freedom involved, an exhaustive exploration of an ambiguity patch is time consuming. We employed two methods. The first involves choosing beforehand a set of $N_{\mathrm{c}} + 1$ waves, where N_{c} is the number of constrained inelasticities. One of these inelasticities is chosen to belong to I^{f}, and it is changed by a standard increment, ΔI^{f}. The remaining N_{c} waves belong to the constrained set, the I_l^{c} being changed automatically as the iteration proceeds. When convergence has been reached, to a preset criterion of accuracy, the program uses the results of the old iteration as a starting point for a new one. Singularities, such as failure of convergence of the iteration or exit of one I_l^{c} from the allowed interval $[0,\frac{1}{4}]$, are dealt with automatically,

Table 1
Phase shifts in radians for the original (SACLAY) solution, and for the five ambiguities A–E, $|l| \leq 6$.

l	Saclay	A	B	C	D	E
0	−1.07950000	−1.03157677	−1.06292061	−1.02517096	−1.03261526	−1.08588038
1−	−0.45766000	−0.49051896	−0.47102036	−0.49488099	−0.43522720	−0.79060551
1+	0.17291000	0.18515922	0.17372475	0.18747786	0.17717161	0.17523082
2−	−0.19645000	−0.18879372	−0.19233731	−0.18806332	−0.19138531	−0.18489492
2+	−0.08252000	−0.05644697	−0.06872898	−0.05396849	−0.04797109	−0.12511294
3−	−0.14807000	−0.15614125	−0.15252225	−0.15689490	−0.15158169	−0.17993989
3+	−0.26992000	−0.31403630	−0.29492276	−0.31746639	−0.31719395	−0.23353944
4−	−0.00838000	−0.00241907	−0.00511625	−0.00190546	0.00033625	−0.02650831
4+	−0.01494000	−0.00137852	−0.00704611	−0.00020612	−0.00432779	0.00464090
5−	−0.01534000	−0.01720294	−0.01721196	−0.01717273	−0.01840238	−0.01557665
5+	0.02246000	0.02139755	0.01964608	0.02178429	0.02055720	0.01908527
6−	−0.01210000	−0.01100400	−0.01103432	−0.01099573	−0.01049899	−0.01244348
6+	−0.00035000	0.00249166	0.00172365	0.00264940	0.00256813	−0.00043269

by a new choice of l_l^f from the chosen set of $N_c + 1$ waves. A complete run consists of many such iterations, and an exhaustive search of the ambiguity patch at a given energy entails the combination of runs, corresponding to different choices of the $N_c + 1$ waves, each run starting from the same solution, labelled SACLAY 74, that was used in ref. [2]. Our second method, which is quicker but less thorough, involves performing a small number of convergent iterations with a given choice of the $N_c + 1$ waves, and then randomly choosing a new set of $N_c + 1$ waves, and so on.

In table 1 we display the phase shifts, δ_l, $|l| \leq 6$, corresponding to the SACLAY 1974 analysis at 1997 MeV, together with five examples from the continuum ambiguity patch, labelled A, B, C, D and E. Notice that the S wave varies from −1.025 in ambiguity C to −1.086 in E. Other waves also show remarkable variations.

In table 2 we give the peripheral waves up to $|l| = 15$. In the SACLAY solution the peripheral waves

Table 2
Peripheral phase shifts for the ambiguities, $7 \leq |l| \leq 15$.

l	A	B	C	D	E
7−	−0.613715E−3	−0.572195E−3	−0.620004E−3	−0.384477E−3	−0.278481E−2
7+	−0.105872E−2	−0.104284E−2	−0.106797E−2	−0.956515E−3	−0.213891E−2
8−	0.118879E−3	0.226811E−3	0.989948E−4	0.115505E−3	0.295979E−3
8+	0.186283E−3	0.424444E−3	0.143540E−3	0.251168E−3	0.797510E−3
9−	−0.765741E−4	−0.102283E−3	−0.735822E−4	−0.643817E−4	−0.101274E−3
9+	−0.271206E−3	−0.234074E−3	−0.280942E−3	−0.215459E−3	−0.475461E−3
10−	0.303303E−4	0.416201E−4	0.283124E−4	0.250891E−4	0.372354E−4
10+	0.552207E−4	0.863934E−4	0.503284E−4	0.430987E−4	0.238190E−3
11−	0.485246E−6	−0.107479E−4	0.226394E−5	−0.171163E−5	0.217584E−4
11+	−0.857269E−5	−0.271043E−4	−0.578939E−5	−0.102776E−4	−0.326548E−4
12−	0.295495E−5	0.282935E−5	0.306519E−5	0.228912E−5	−0.125662E−4
12+	0.173282E−4	0.116824E−4	0.183533E−4	0.119821E−4	0.891022E−5
13−	0.233992E−6	0.206825E−6	0.337768E−6	0.424562E−7	0.607208E−5
13+	0.104683E−5	−0.110768E−5	0.149860E−5	0.865861E−6	0.433075E−5
14−	−0.923327E−6	−0.974677E−6	−0.921285E−6	−0.626516E−6	−0.461998E−5
14+	−0.284456E−6	−0.735462E−6	−0.210846E−6	−0.744885E−7	−0.895515E−5
15−	0.988929E−9	0.541937E−6	−0.643705E−7	0.336825E−7	0.658239E−6
15+	−0.714773E−6	0.425963E−6	−0.842720E−6	−0.448429E−6	0.245461E−5

Table 3
Elasticities for the original (SACLAY) solution, and for the ambiguities, $|l| \leqslant 6$.

l	Saclay	A	B	C	D	E
0	0.87200000	0.87984374	0.86560358	0.88277464	0.89082395	0.69376121
1−	0.41210000	0.41205399	0.40825542	0.41316796	0.41307927	0.43355358
1+	0.53730000	0.49423438	0.51397556	0.49029000	0.46526212	0.68545946
2−	0.80070000	0.79872100	0.80240225	0.79777048	0.81075639	0.73220241
2+	0.77520000	0.78333569	0.78291766	0.78320243	0.79321205	0.71831142
3−	0.79330000	0.79745533	0.79391435	0.79823851	0.80026912	0.76628973
3+	0.37780000	0.41157388	0.38882950	0.41656083	0.41229757	0.37780000
4−	1.00000000	0.98956610	0.99812686	0.98767888	1.00000000	0.94192807
4+	0.86520000	0.86520000	0.86520000	0.86520000	0.84779759	0.96786547
5−	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5+	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
6−	0.99200000	0.99200000	0.99200000	0.99200000	0.99200000	0.99200000
6+	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

$(7 \leqslant |l| \leqslant 15)$ vanish, but in the ambiguities they have been retained. Their values range from $\sim 10^{-2}$ to $\sim 10^{-7}$, but, as we have remarked above, there is no justification for forcing these waves to be zero, given the exponential convergence of the partial wave series caused by our method of constructing the ambiguities. This tail of peripheral waves is of course essential in keeping $d\sigma/d\Omega$, P and σ_{tot} constant, despite the considerable changes in the lower waves of the ambiguities in table 1.

A glance at table 2 shows that the values of the high waves vary enormously from one ambiguity to another. Nevertheless, careful observation reveals certain regularities. In particular, for each ambiguity one finds that the l^+ waves oscillate in sign as l runs from 6 to 15, the effect being largest for ambiguity E, which differs most from the SACLAY 74 starting point. A similar, but less clear signal is seen in the l^- waves. This effect is due to singularities outside the small Martin ellipse, which are generated by our method. Such singularities are of course present in the particular process under consideration: the nucleon pole in the u-channel dominates the physical peripheral waves, and causes oscillations similar to what we observe. Although we do not control the singularities generated, the high partial waves of table 2 do mimic the correct oscillating behaviour. Notice also that the high partial waves for ambiguity E are (in general)

Table 4
Differential and total cross sections for the SACLAY solution, and the differences between them and those for the ambiguities. The differences should be multiplied by 10^{-5}.

	$\cos\theta$	Saclay 1974	A	B	C	D	E
differential	1.0	14.037900	1.8	1.9	2.0	1.3	1.1
	0.8	2.496832	−0.2	−0.2	−0.2	−0.1	−0.2
	0.6	0.239106	−0.1	0.0	−0.1	0.0	−0.1
	0.4	0.143617	0.0	0.0	0.0	0.0	0.0
	0.2	0.273288	0.0	0.0	0.0	0.0	0.0
	0.0	0.404681	0.0	0.0	0.0	0.0	0.1
	−0.2	0.646801	−0.1	−0.2	−0.1	−0.1	−0.2
	−0.4	0.833686	0.2	0.3	0.2	0.1	0.3
	−0.6	0.619728	−0.1	−0.2	−0.1	−0.1	0.2
	−0.8	0.201703	−0.2	−0.3	−0.2	−0.1	−0.4
	−1.0	0.586655	0.1	0.5	0.1	0.0	−0.3
total		35.494289	0.0	0.0	0.0	0.0	0.0

Table 5
Polarization for the SACLAY solution and the corresponding differences, which should again be multiplied by 10^{-5}.

$\cos\theta$	Saclay 1974	A	B	C	D	E
0.8	−0.450793	0.0	0.0	0.0	0.0	0.0
0.6	−0.781505	0.2	0.1	0.2	0.2	0.7
0.4	−0.693519	0.5	0.4	0.5	0.3	0.7
0.2	−0.556566	−0.5	−0.4	−0.5	−0.3	−0.9
0.0	−0.347255	0.4	0.4	0.4	0.3	0.8
−0.2	−0.092772	−0.2	−0.2	−0.2	−0.1	−0.5
−0.4	−0.057478	0.1	0.1	0.1	0.1	0.4
−0.6	−0.262535	−0.1	−0.1	−0.1	−0.1	−0.7
−0.8	−0.837043	−0.3	−0.4	−0.3	−0.2	−0.6

larger than those for the ambiguities A–D. In this case the new singularities have come close to the Martin ellipse. The peripheral waves are thus larger, and also oscillate faster.

In table 3 we display, for $|l| \leqslant 6$, the values of the elasticities, η_l, related to the I_l by

$$I_l = \tfrac{1}{4}(1 - \eta_l^2), \tag{5}$$

for the SACLAY solution and for our ambiguities. Here the η_l are restricted to lie in the interval [0,1], with $\eta_l = 1$ or $I_l = 0$ corresponding to complete elasticity. As can be seen from the table, for $|l| \leqslant 6$, most elasticities have been changed, and in some cases η_0 and $\eta_{\pm 1}$ deviate by as much as −13% – +28% from

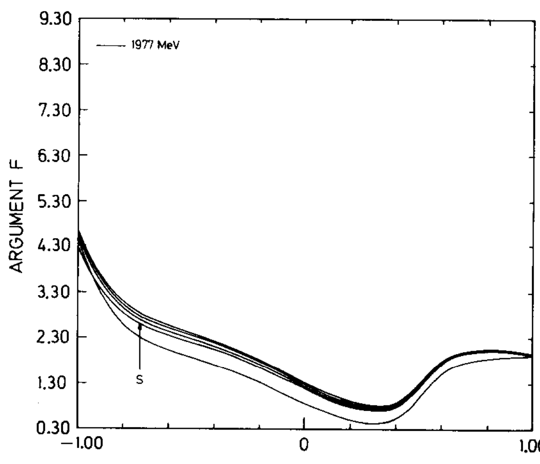

Fig. 1. The argument of $F(\zeta)$, as a function of $\cos\theta$, for the original SACLAY solution (labelled S) and for the ambiguities.

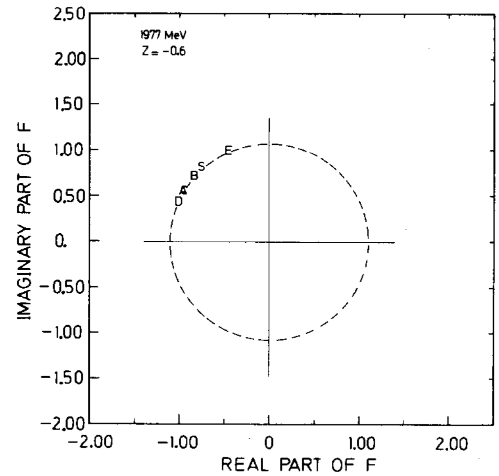

Fig. 2. Argand plot of $F(\zeta)$ at $\cos\theta = -0.6$ for the SACLAY solution (S), and for the ambiguities (A–E). The dotted circle corresponds to $|F(\zeta)| = |F^S(\zeta)|$.

the SACLAY values. The higher waves ($7 \leqslant |l| \leqslant 15$) in all the ambiguities were completely elastic. Some of these elasticities could have been altered in runs of our program; but the effect on the lower partial waves would have been very small. It is the freedom that the high-l tail has to wag about within the allowed exponential bound that gives rise to the continuum ambiguity.

A serious conceptual error has been made by those who insist that the very high partial waves, which are not well determined by the data, should be set equal

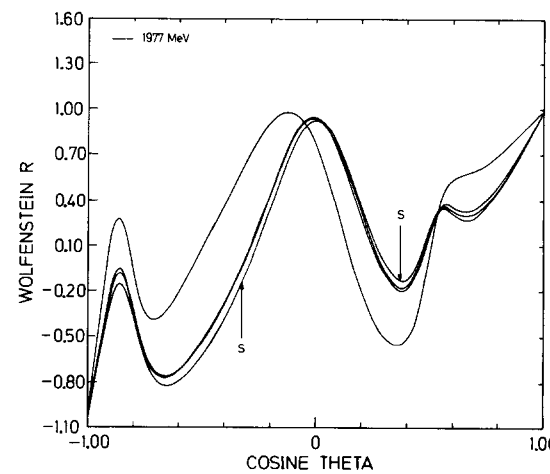

Fig. 3. The Wolfenstein parameter, R, for the SALCAY solution labelled S and for the ambiguities.

to zero. Such a procedure gives only one point in the continuum patch; and this may well lie, in particular cases, quite far from the physically correct solution, even in so far as the S and P waves are concerned.

Finally, we examine the degree of accuracy with which the ambiguous solutions reproduce the experimental data. In table 4 we give the differential cross section, $d\sigma/d\Omega$, as a function of $\cos\theta$, and the total cross section, σ_{tot}, for the SACLAY solution, together with the deviations from these values for the five ambiguities. The polarization is displayed in a similar way in table 5. We see that in most cases the total cross section changes by less than one part in 10^5, and the differential cross sections and polarizations by never more than ~ 2 parts in 10^5, and usually by less.

Summarizing, we have created an uncertainty of the order of 5% in the magnitude of the lowest partial waves. Fig. 1 shows the arguments of the SACLAY solution and the five ambiguities, while fig. 2 plots the positions of the different cases in the complex plane of $F(\zeta)$ (the scattering amplitude in terms of the Barrelet variable [9], ζ). As can be seen, $|F(\zeta)|$ is constant. Finally, in fig. 3, we plot the Wolfenstein parameter R, which is experimentally, but not easily, accessible. The ambiguity patch can be made smaller by measuring R, but a phase ambiguity always remains [10].

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